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ANALYSIS OF CIRCULAR AND RECTANGULAR APERTURES IN A CIRCULAR WAVEGUIDE

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ABSTRACT

An analysis has been carried out to determine the discontinuity susceptance of a circular or rectangular aperture in the transverse plane of a circular waveguide. A closed-form solution has been derived with good agreement as compared with experimental results. The results should have many applications in the design of circular waveguide components and cylindrical cavity-backed phased arrays.

I. INTRODUCTION

Diffraction of incident electromagentic field by a small hole was first investigated by Bethe [1] for small apertures. Later, Collin [2] summarized Bethe's approach and defined coupling coefficients which make it possible to determine the electrical properties of the aperture. The electrical properties of the aperture, such as susceptance, are important in the design of coupled cavity resonant filters [3, 4], directional couplers, and conformal cavity-backed aperture antennas [5].

Apertures in rectangular waveguide have been thoroughly investigated [6-8], but apertures in circular waveguides have only received sparse attention [1]. A truly comprehensive study of different aperture shapes embedded in a circular waveguide does not exist. The purpose of this paper is to study various apertures and slot antennas in a circular waveguide.

This paper reports an analysis based on the scattered amplitude calculation and aperture coupling theory. A closed-form, easy to use formula was derived for a general aperture discontinuity in the transverse plane of a circular waveguide. Example solutions for a small circular aperture and a resonant rectangular aperture were given. The calculated results agree very well with the experiments. The analysis should have many applications in the design of circular waveguide components and circular-waveguide backed conformal phased arrays.

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II. ANALYSIS AND CLOSED-FORM SOLUTION

A closed-form solution has been derived for both circular and rectangular apertures in a circular waveguide. The solution was based on scattered amplitude calculation and the aperture coupling theory. The results can be used to predict the resonant frequency of an aperture discontinuity or a slot antenna fed by a circular waveguide cavity.

Figure 1 shows a general aperture discontinuity in the transverse plane of a circular waveguide. The scattered amplitude coefficient for the dominant mode incident (TE_{11}) can be calculated by

$$C_1^+ = \frac{j2\alpha_m (p'_{11})^2}{\lambda_g a^2 [(p'_{11})^2 - 1] J_1^2(k_c a)} \qquad (1)$$

where p'_{11} is the first root of the derivative of the Bessel function, $J'_1(k_c r) = 0$. α_m is the magnetic polarizability. λ_g is guide wavelength and k_c is the cutoff wave number.

The reflection coefficient Γ is given by

$$\Gamma = T - 1 = C_1^+ - 1 \qquad (2)$$

The effect of an infinitely thin transverse aperture on the dominant mode field of a waveguide can be described by a shunt susceptance across the waveguide as shown in Figure 2.

For a small aperture, the normalized susceptance $\bar{B}$ is big. The input reflection coefficient can be approximated by

$$\Gamma = -\left(\frac{2 + j\bar{B}}{j\bar{B}}\right)^{-1} = -\left(1 + \frac{2}{j\bar{B}}\right)^{-1} \approx -1 - j\frac{2}{\bar{B}} \qquad (3)$$

Therefore, by comparing Eqs (2) and (3), the normalized susceptance is

$$\bar{B} = -\frac{\lambda_g a^2 [(p'_{11})^2 - 1]\, J_1^2(k_c a)}{\alpha_m (p'_{11})^2} \qquad (4)$$

III. SMALL CIRCULAR APERTURE

Substituting $p'_{11} = 1.841$ and $J_1(p'_{11}) = 0.58$ into Eq (4), and using α_m for a small circular aperture given by [1, 7], we have

$$\bar{B} = -\frac{0.178\lambda_g a^2}{r_0^3} \quad (5)$$

Figure 3 shows the theoretical results compared with the experimental results for a circular aperture of radius equal to 0.175 cm in a circular waveguide with a radius of 1 cm. The agreement is fairly good except at frequencies near the cutoff frequency. The resonant frequency is well above the operating frequency range of the circular waveguide. The experimental results were obtained by using a HP 8510 network analyzer.

By examining Eq (5) it is evident that the susceptance will vary inversely to the cube of the apertures radius r_0. It can be seen that a minor error in the radius measurement could have a dramatic effect on the susceptance.

IV. NARROW RECTANGULAR APERTURE

The small aperture approximation discussed earlier can be modified and adapted to an aperture whose resonant frequency is close to the operating frequency of the waveguide. Foster's reactance theorem [4] is implemented to derive a frequency correction term for the normalized susceptance of the aperture. Applying this theory to a resonant rectangular aperture with a small aspect ratio (i.e. W/L << 1) in a circular waveguide, the susceptance can be obtained with a simple closed-form expression. The small aspect ratio is necessary so that the incident fields are not significantly perturbed and the frequency correction factor can be applied. Using this method, a theory is devised to determine the susceptance of a transverse aperture whose resonant frequency is in the operating frequency range of the circular waveguide in which it is embedded without having to resort to numerical solutions.

Figure 4 shows the rectangular aperture with length L and width W. The magnetic polarizability of a rectangular aperture was derived by McDonald and De Smedt [9-11] with the following form:

$$\alpha_m = \frac{0.132}{\ell_n(1 + 0.66/\alpha)} L^3 \quad (6)$$

where α is the aspect ratio defined by W/L.

By considering a second order quadratic approximately, Eq (4) can be modified by including a $(1 - \frac{f^2}{f_m^2})$ factor derived from Foster reactance theorem [12]. The normalized susceptance is

$$\bar{B} = -\frac{\lambda_g a^2[(p'_{11})^2 - 1]J_1^2(k_c a)}{\alpha_m (p'_{11})^2}\left(1 - \frac{f^2}{f_m^2}\right) \quad (7)$$

f_m is the resonant frequency of the aperture which can be approximated from its cutoff frequency or determined experimentally.

For example, if an aperture resonant frequency of 11.55 GHz is assumed, the resonant length L of the rectangular aperture is approximately equal to 12 mm. Using an aspect ratio of 6, the measured and calculated normalized susceptances as a function of frequency are shown in Figure 5 for comparison. The agreement is very good.

V. SLOT ANTENNAS FED BY CIRCULAR WAVEGUIDES

The analysis for apertures can be applied to calculate the resonance frequency of a slot antenna fed by a circular waveguide. The normalized susceptance obtained from the transverse aperture embedded in a waveguide is halved to determine the susceptance of the resonant slot antenna [8].

VI. CONCLUSIONS

In conclusion, a closed-form expression for the discontinuity susceptance of an aperture in the transverse plane of a circular waveguide has been derived. The theoretical results agree very well with the experiments for both a small circular aperture and a resonant rectangular aperture. The results are useful for circular waveguide component and antenna design.

VII. ACKNOWLEDGEMENTS

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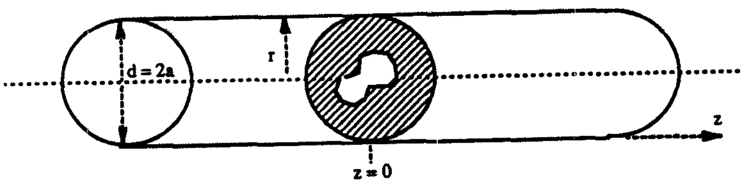

Fig 1 Circular waveguide with an arbitrary aperture at a transverse conducting wall

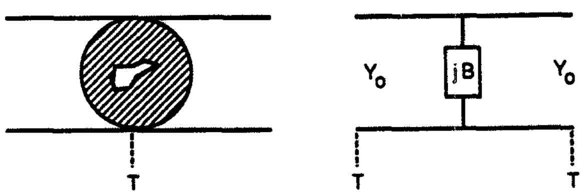

Fig 2 Equivalent circuit of the aperture on the transverse conducting wall

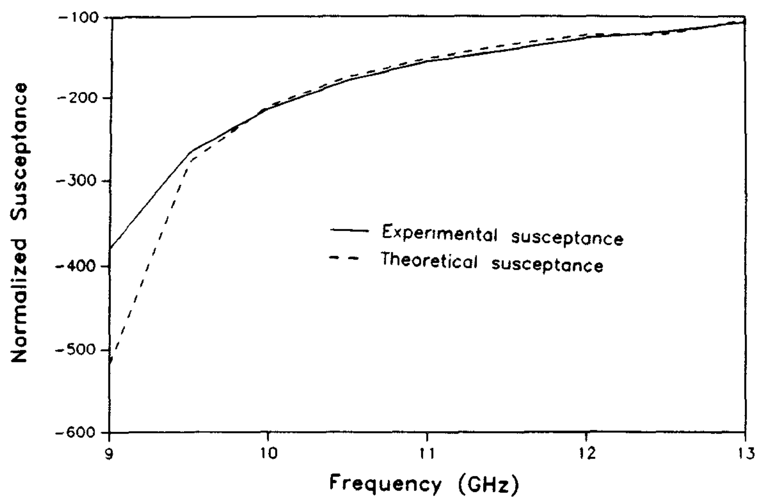

Fig 3 Theoretical and experimental susceptance results for a circular aperture on the transverse plane of a circular waveguide

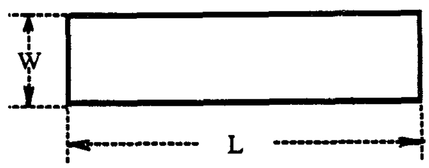

Fig 4 Rectangular aperture with length L and width W

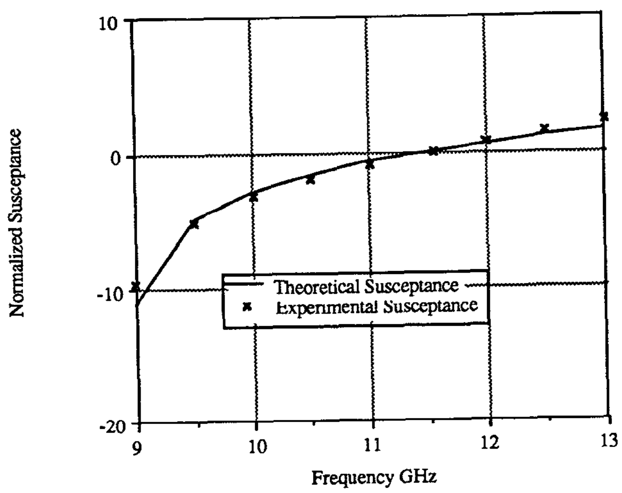

Fig 5 Comparison of theoretical and experimental susceptance for a resonant rectangular aperture on the transverse plane of a circular waveguide with radius of 1 cm.